AD1 revision, the graph library, and graph search

Frej Knutar Lewander

Uppsala University
Sweden

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Outline

1. Python
2. Data Structures revision
3. Graphs
4. Breadth- and Depth-First Search
5. Greedy Algorithms

Why Python

The coding part of the assignments is done in Python. Why have we decided to use python, when

- other languages are faster (Java, C, C++, etc.),
- you haven’t used Python before,
- Python is not your language of choice, etc.

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The ideas and mindset gained from this course are independent of the programming language they are applied to.

Python is just “a means to an end”.

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- accessing an element at index i ($0 \leq i < n$): $\Theta(?)$
- adding an element: $\Theta(?)$
- removing an element: $\Theta(?)$
- checking if A contains the element e: $\Theta(?)$
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- adding an element: $\Theta(n)$
- removing an element: $\Theta(n)$
- checking if A contains the element e: $\Theta(n)$
- adding an element at index n: $\Theta(1)$

Arrays

Arrays tend to be the default data structure of choice.
Here are some (non-exhaustive) tips on when to use an array:

- checking if an element exists in the array does not worsen the time complexity;
- the order of the elements matter; or
- the elements can be accessed by their (distinct) index fast (does not worsen the time complexity).

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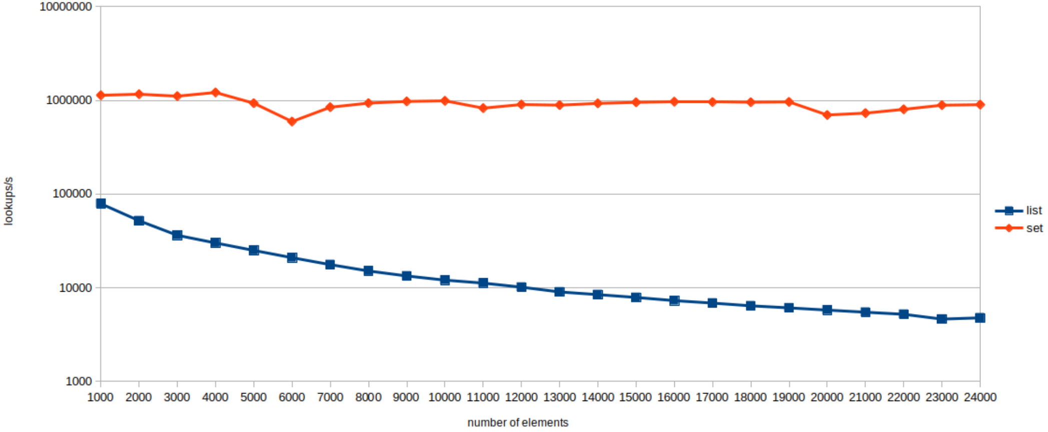
Sets

Here are some (non-exhaustive) tips on when to use a set:

- checking if an element exists in the set is integral to the task at hand;
- duplicate elements are not allowed; or
- each element can *not* be accessed by a (distinct) index.

Comparison (logarithmic)



Hash Tables

A hash table H (dictionary in Python) of *cardinality* n holds n elements, each identified by a distinct key:

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Given a hash table H with cardinality n, the average case time complexity for:

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Hash tables

Hash tables tend to be the data structure of choice for simpler nested data. Here are some (non-exhaustive) tips on when to use a hash table:

- a set with some extra data is required or
- the data is simple, and creating classes (or other custom data structures) for the data is “overkill”.

Graphs

The graph.py Library (1)

A graph is represented similarly to the Adjacency list: the edges are represented as a dictionary with nodes as keys and sets of nodes as elements.

For a graph $G = (V, E)$:

Space complexity:	$\Theta(n + m)$
checking if G contains an edge (u, v):	$\Theta(1)$
Identifying all edges:	$\Theta(m)$

The graph.py Library (2)

Given a graph `G=(V,E)` the properties you need for the first assignment are:

- `G.edges` is a (duplicate free) unsorted list containing all edges `E` and
- `G.nodes` is a (duplicate free) unsorted list containing all vertices `V`

The graph.py Library (3)

Some graphs, $G = (V, E)$, have additional properties for the edges. In the graph library, an edge (u, v) can have:

- a *flow* that is integer or `None`, (typically) denoting the amount of some commodity that travels over (u, v);
- a *capacity* that is integer or `None`, (typically) denoting the maximum amount of some commodity that can travel over (u, v); and
- a *weight* that is integer or `None`, (typically) denoting the cost of including or traversing (u, v).

The graph.py Library (4)

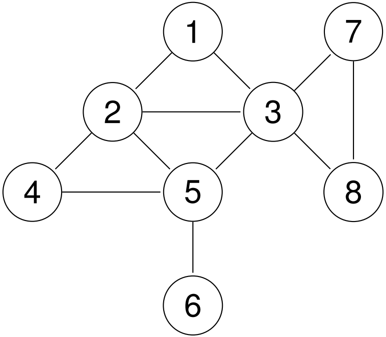
Given a graph `G` and an edge `(u,v)`, the flow, capacity, and weight of `(u,v)` be accessed or modified by the methods:

- `G.flow(u, v)` and `G.set_flow(u, v, f)`;
- `G.capacity(u, v)` and `G.set_capacity(u, v, c)`; and
- `G.weight(u, v)` and `G.set_weight(u, v, w)`,

respectively for some $\mathtt{f}, \mathtt{c}, \mathtt{w} \in \{\mathtt{None}\} \cup \mathbb{Z}$.

Breadth- and Depth-First Search

Given an unordered graph $G = (V, E)$, starting at node s, is there a path in G from s to t?



BFS – pseudocode

```
algorithm BFS($G, s, t$)
    $Visited \leftarrow \{s\}$
    $Q \leftarrow$ an empty queue
    ENQUEUE($Q, s$)
    while $|Q| > 0$ do                                   // Variant: ...
        $u \leftarrow$ DEQUEUE($Q$)
        if $u = t$ then
            return true
        for each $(w, v) \in E$ where $w = u$ do         // Variant: ...
            if $v \notin Visited$ then
                ENQUEUE($Q, v$)
                $Visited \leftarrow Visited \cup \{v\}$
    return false
```

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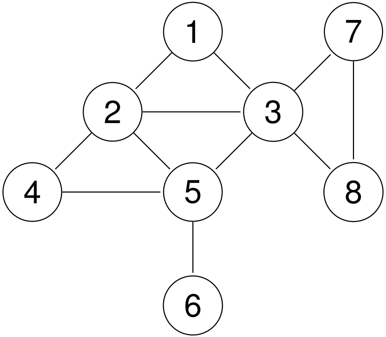
algorithm BFS($G, s, t$)
    $Visited \leftarrow \{s\}$
    $Q \leftarrow$ an empty queue
    ENQUEUE($Q, s$)
    while $|Q| > 0$ do                                   // Variant: $n - |Visited|$
        $u \leftarrow$ DEQUEUE($Q$)
        if $u = t$ then
            return true
        for each $(w, v) \in E$ where $w = u$ do          // Variant: $degree(u) - \#$iterations
            if $v \notin Visited$ then
                ENQUEUE($Q, v$)
                $Visited \leftarrow Visited \cup \{v\}$
    return false
```

BFS – Python code

```python
def bfs(graph: Graph, s:str, t:str) -> bool:
    visited = {s}
    queue = deque()
    queue.append(s)
    while len(queue) > 0:
        u = queue.popleft()
        if u == t:
            return True
        for v in graph.neighbors(u):
            if v not in visited:
                queue.append(v)
                visited.add(v)
    return False
```

BFS

With $s = 1$ and $t = 6$, what set of nodes will be visited during $\textsc{Bfs}(G, s, t)$? Can the set differ between runs?



DFS – pseudocode

```
**algorithm** DFS($G, s, t$)
    $Visited \leftarrow \{s\}$
    $S \leftarrow$ an empty stack
    PUSH($S, s$)
    **while** $|S| > 0$ **do**                                   // Variant: $n - |Visited|$
        $u \leftarrow$ POP($S$)
        **if** $u = t$ **then**
            **return true**
        **for each** $(w, v) \in E$ **where** $w = u$ **do**      // Variant: $degree(u) - \#$iterations
            **if** $v \notin Visited$ **then**
                PUSH($S, v$)
                $Visited \leftarrow Visited \cup \{v\}$
    **return false**
```

DFS – pseudocode

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algorithm DFS($G, s, t$)
    $Visited \leftarrow \{s\}$
    $S \leftarrow$ an empty stack
    PUSH($S, s$)
    while $|S| > 0$ do                                          // Variant: $n - |Visited|$
        $u \leftarrow$ POP($S$)
        if $u = t$ then
            return true
        for each $(w, v) \in E$ where $w = u$ do                // Variant: $degree(u) - \#$iterations
            if $v \notin Visited$ then
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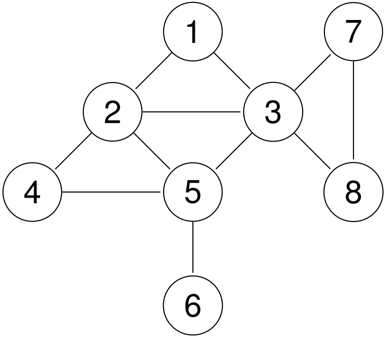

DFS – Python code

```python
def dfs(graph: Graph, s:str, t:str) -> bool:
  visited = {s}
  stack = []
  stack.append(s)
  while len(stack) > 0:
    u = stack.pop()
    if u == t:
      return True
    for v in graph.neighbors(u):
      if v not in visited:
        stack.append(v)
        visited.add(v)
  return False
```

DFS – Python code (contd)

Why can we use a list as a stack without worsening the time complexity?

DFS – Python code (contd)

Why can we use a list as a stack without worsening the time complexity?

Adding an element to the end of a list or array has (amortised) time complexity $\Theta(1)$. We discuss amortised time complexity in AD3.

We could also initialize a list with *n* elements and keep a “stack pointer”:

DFS with Stack Pointer – Python code

```python
def dfs_sp(graph: Graph, s:str, t:str) -> bool:
  visited = {s}
  stack = [s] + ([None] * (len(graph.nodes) - 1))
  top = 0 # stack[top] = top element of stack
  while top >= 0:
    u = stack[top]
    top -= 1
    if u == t:
      return True
    for v in graph.neighbors(u):
      if v not in visited:
        top += 1
        stack[top] = v
        visited.add(v)
  return False
```

Greedy Algorithms

A greedy algorithm makes the locally optimal choice in every step.

Greedy Algorithms - Coin Change

We are to return the (cash) change 19 SEK using the fewest amount of coins (coin denominations are 10 SEK, 5 SEK, 2 SEK, and 1 SEK).

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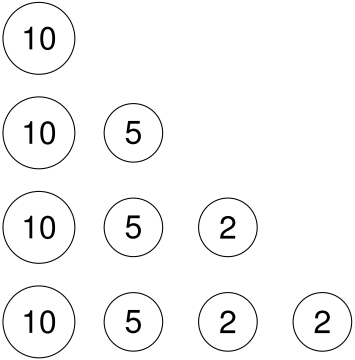
$$19 - 10 = 9$$

$$9 - 5 = 4$$

$$4 - 2 = 2$$

$$2 - 2 = 0$$



Greedy Algorithms

When are greedy algorithms good?

Greedy Algorithms

When are greedy algorithms good?

When selecting the locally optimal choice in every step yields a sufficiently good global solution.

In the coin example, the global solution was optimal, but that is not guaranteed.